SCRITTO del 06/09/2019

1) $$\lim_{x\to 0} \frac{e^{\sin x - x^2} - \cos x - x + \log(1+2x^3)}{\log(x+2)\left(\sqrt{1+x^3}-1\right)}$$

Num:

- $$\boxed{e^{\sin x - x^2}} = e^{x - \frac{x^3}{6} - x^2 + o(x^3)}$$

$$= 1 + x - \cancel{\frac{x^3}{6}} - x^2 + \frac{1}{2}(x-x^2)^2 + \cancel{\frac{1}{6}x^3} + o(x^3)$$

$$= 1 + x - x^2 + \frac{1}{2}x^2 - x^3 + o(x^3)$$

$$= \boxed{1 + x - \frac{x^2}{2} - x^3 + o(x^3)}$$ per $x \to 0$

- $$-\cos x = -1 + \frac{1}{2}x^2 + o(x^3)$$
- $$\log(1+2x^3) = 2x^3 + o(x^3)$$

Quindi: $$\underline{\text{Num}} = \cancel{1} + \cancel{x} - \cancel{\frac{x^2}{2}} - x^3 - \cancel{1} + \cancel{\frac{1}{2}x^2} - \cancel{x} + 2x^3 + o(x^3)$$

$$= \underline{x^3 + o(x^3)}$$ per $x \to 0$

Den: $\cdot$ $\log(x+2) \to \log 2$ per $x \to 0$

$\cdot$ $\sqrt{1+x^3} - 1 = 1 + \frac{1}{2}x^3 - 1 + o(x^3) = \frac{1}{2}x^3 + o(x^3)$

Conclusione:

$$\lim_{x\to 0} \frac{\text{Num}}{\text{Den}} = \lim_{x\to 0} \frac{x^3}{(\log 2)\frac{1}{2}x^3} = \boxed{\frac{2}{\log 2}}$$

2) $f: \mathbb{R} \to \mathbb{R}$, $g: \mathbb{R} \to \mathbb{R}$ $\quad g(x) = f(x^2 + f(x) \sin x)$ (2)

$$g'(x) = f'(x^2 + f(x) \operatorname{sen} x) \cdot [2x + f'(x) \sin x + f(x) \cos x]$$

$$\to \boxed{g'(\pi)} = f'(\pi^2) \cdot [2\pi + \cancel{f'(\pi) \cdot \emptyset} + f(\pi) \cdot 1]$$

$$= 1 \cdot [2\pi + \emptyset] = \boxed{2\pi}$$

3) $$\int_1^2 x^2 (\log x)^2 \, dx = \int_1^2 \frac{x^3}{x} (\log x)^2 dx$$

pongo $\log x = t \to x = e^t$
$dt = \frac{1}{x} dx$ $\quad \to \quad = \int_0^{\log 2} e^{3t} \cdot t^2 \, dt$

$\underset{\text{per parti}}{=} \left[\frac{1}{3} e^{3t} \cdot t^2 \right]_0^{\log 2} - \frac{1}{3} \int_0^{\log 2} e^{3t} \cdot 2t \, dt$

$\underset{\text{per parti}}{=} \frac{1}{3} \cdot 8 \cdot (\log 2)^2 - \frac{2}{9} \int_0^{\log 2} e^{3t} \, dt =$

$= \frac{8}{3} (\log 2)^2 - \frac{2}{27} \left[e^{3t} \right]_0^{\log 2} =$

$= \frac{8}{3} (\log 2)^2 - \frac{2}{27} \cdot 8 - \frac{2}{27}$

(4) $$\int_2^{+\infty} \overbrace{\frac{e^{2-x}}{(x-2)^{\alpha}+\sin^2(x-2)}}^{f(x)}\,dx = \underbrace{\int_2^3 f(x)\,dx}_{II} + \underbrace{\int_3^{+\infty} f(x)\,dx}_{I}$$ (3)

- I_α $x \sim +\infty$ l'integrale converge $\forall \alpha \in \mathbb{R}^+$ (per la presenza di e^{2-x})

- II_α $x \sim 2 \rightarrow f(x) \sim \dfrac{1}{(x-2)^{\alpha}+\cancel{}(x-2)^2}$

$$\Rightarrow \int_2^3 f(x)\,dx < +\infty \iff \int_2^3 \frac{1}{(x-2)^{\alpha}+(x-2)^2}\,dx < +\infty$$

$$\iff \boxed{0<\alpha<1}$$

Conclusione: l'integrale converge per $\boxed{0<\alpha<1}$

(5) Risolvere $\begin{cases} y''-4y'+4y = \sin t + \cos t \\ y(0)=0 \\ y'(0)=0 \end{cases}$

Cerco la soluz. generale dell'eq. omogenea:

EQ. CARATTERISTICA: $\lambda^2-4\lambda+4=0 \iff (\lambda-2)^2=0$

$\rightarrow \lambda = 2$ con molteplicità 2.

sol. gener. $\rightarrow$ $\boxed{y_{om}(t) = c_1 e^{2t} + c_2 t e^{2t}}$

Cerco soluz. particolare dell'eq. non omogenea della forma

$$\tilde{y}(t) = a\,\mathrm{sen}\,t + b\cos t$$

$$\tilde{y}'(t) = a\cos t - b\,\mathrm{sen}\,t$$

$$\tilde{y}''(t) = -a\,\mathrm{sen}\,t - b\cos t$$

$$\Rightarrow \tilde{y}'' - 4\tilde{y}' + 4y = \underline{-a\,\mathrm{sen}\,t} - \underline{b\cos t} - 4(a\cos t - b\,\mathrm{sen}\,t) + 4(\underline{a\,\mathrm{sen}\,t} + \underline{b\cos t})$$

$$= 3a\,\mathrm{sen}\,t + 3b\cos t - 4a\cos t + 4b\,\mathrm{sen}\,t$$

$$= \mathrm{sen}\,t\,(3a + 4b) + \cos t\,(3b - 4a)$$

Imponendo che $\tilde{y}$ sia soluzione, ottengo:

$$\begin{cases} 3a + 4b = 1 \\ 3b - 4a = 1 \end{cases} \Leftrightarrow \begin{cases} a = \dfrac{1-4b}{3} \\ 3b - \dfrac{4}{3} + \dfrac{16}{3}b = 1 \end{cases}$$

$$\begin{cases} - \\ \dfrac{25}{3}b = \dfrac{7}{3} \end{cases} \Rightarrow \begin{cases} a = \dfrac{1 - \frac{28}{25}}{3} \\ b = \dfrac{7}{25} \end{cases} \Rightarrow \begin{cases} a = -\dfrac{1}{25} \\ b = \dfrac{7}{25} \end{cases}$$

$$\Rightarrow \boxed{\tilde{y}(t) = -\frac{1}{25}\,\mathrm{sen}\,t + \frac{7}{25}\cos t}$$

Soluz generale della non omogenea: ⑤

$$y(t) = c_1 e^{2t} + c_2 t e^{2t} - \frac{1}{25} \operatorname{sen} t + \frac{7}{25} \cos t$$

Impongo le condizioni iniziali per trovare c_1, c_2:

- $y(0) = c_1 + \frac{7}{25} = 0 \rightarrow \boxed{c_1 = -\frac{7}{25}}$

$$y'(t) = 2c_1 e^{2t} + c_2 e^{2t} + 2c_2 t e^{2t} - \frac{1}{25}\cos t - \frac{7}{25}\operatorname{sen} t$$

$$\rightarrow y'(0) = 2c_1 + c_2 - \frac{1}{25} = 0$$

$$\Rightarrow -\frac{14}{25} + c_2 - \frac{1}{25} = 0 \Rightarrow \boxed{c_2 = \frac{15}{25} = \frac{3}{5}}$$

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⑥ $f(x) = \left| \frac{|x| + 1}{|x| - 1} \right|$

DOMINIO: $|x| \neq 1 \rightarrow x \neq \pm 1 \Rightarrow D = \mathbb{R} \setminus \{-1, 1\}$

LIMITI

$\lim_{x \to \pm\infty} f(x) = +1 \rightarrow$ ASINTOTO ORIZZ. $y = 1$

$\lim_{x \to \pm 1} f(x) = +\infty \rightarrow$ ASINTOTI VERTICALI $x = 1$, $x = -1$

MONOTONIA: Osservo che $f(x) = f(-x)$ [f è PARI] quindi basta studiare il caso $x > 0$:

Se $x > 0$ $f(x) = \left| \frac{x+1}{x-1} \right|$

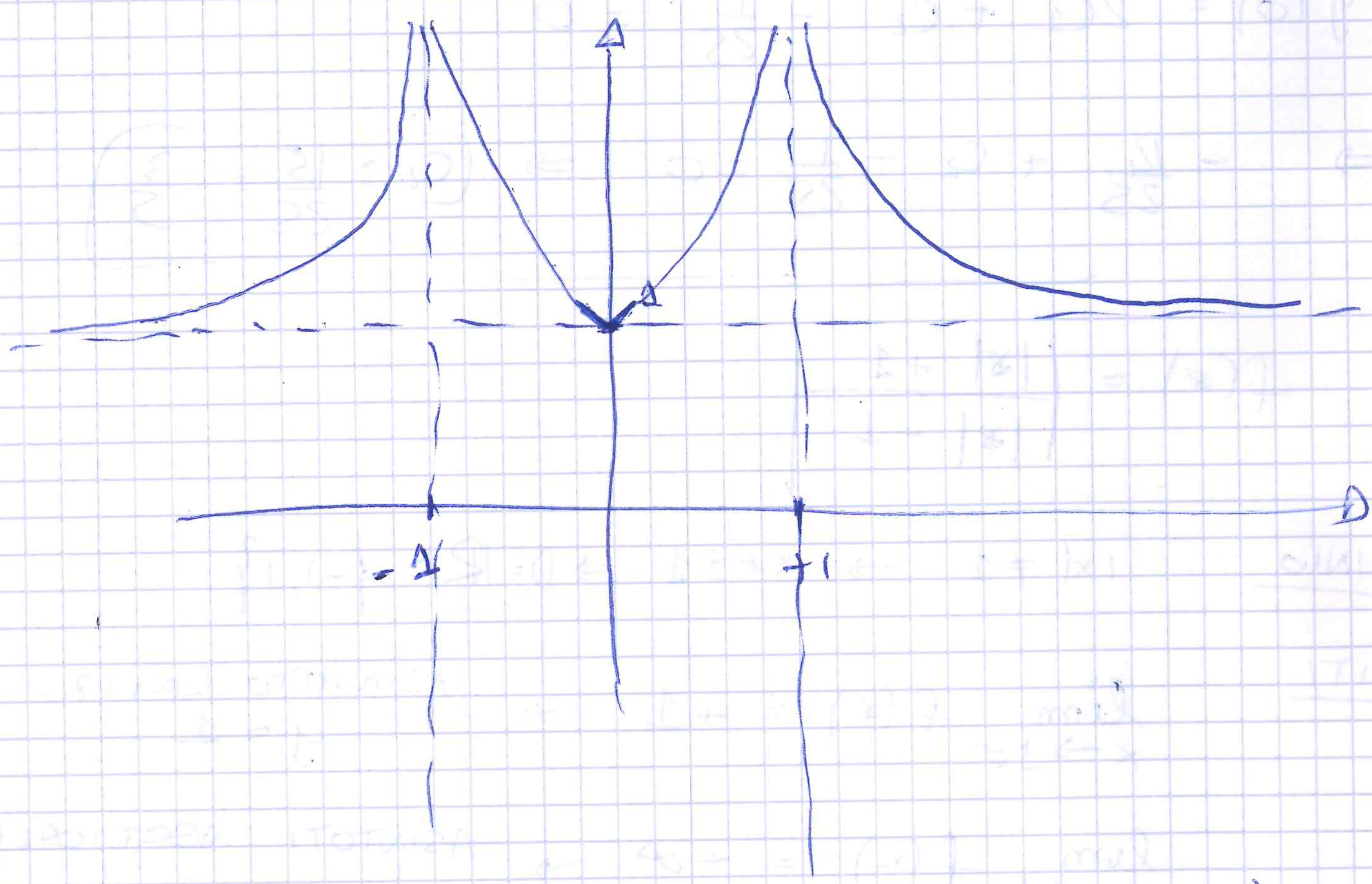
$$f'(x) = \operatorname{sgn}\left(\frac{x+1}{x-1}\right) \cdot \left[\frac{\cancel{x}-1-\cancel{x}-1}{(x-1)^2}\right]$$

$$= -\frac{2}{(x-1)^2} \cdot \operatorname{sgn}\left(\frac{x+1}{x-1}\right)$$

Poiché $x>0 \to x+1>0 \Rightarrow \operatorname{sgn}\left(\frac{x+1}{x-1}\right) = \operatorname{sgn}(x-1)$

$\Rightarrow \quad f'(x) \geq 0 \iff 0<x<1 \quad f \nearrow$

$\quad\; f'(x) \leq 0 \iff x>1 \quad f \searrow$



Ricapitolando: $f \nearrow$ in $(-\infty,-1) \cup (0,1)$

$f \searrow$ in $(-1,0) \cup (1,+\infty)$

- f ha minimo assoluto in $x=0$ e vale $f(0)=1$
- $\sup f = +\infty$

Pti di non derivabilità

f non è derivabile in $x=0$ infatti

Se $x>0$ $\quad f(x) = \left|\frac{x+1}{x-1}\right| =$

$$\to f'(x) = \operatorname{sgn}\left(\frac{x+1}{x-1}\right)\cdot\left(-\frac{2}{(x-1)^2}\right)$$

Se $x<0$ $\quad f(x) = \left|\frac{-x+1}{-x-1}\right|$

$$\to f'(x) = \operatorname{sgn}\left(\frac{1-x}{-x-1}\right)\cdot\left[\frac{+x+1-x+1}{(-x-1)^2}\right]$$

$$= \operatorname{sgn}\left(\frac{1-x}{-x-1}\right)\cdot\frac{2}{(-x-1)^2}$$

$$\lim_{x\to 0^+} f'(x) = 2$$

$$\neq$$

$$\lim_{x\to 0^-} f'(x) = -2$$